Determinants

Property :

The value of determinant will not change if the columns ~~and Rows inter change in corresponding.~~ are changed into correspoing Rows.

Eg

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \quad \text{and } \Delta' = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}$$

proof that $\Delta = \Delta'$

Sol

$$\Delta = 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$\Rightarrow 1(45-48) - 2(36-42) + 3(32-35)$

$\Rightarrow 1(-3) - 2(-6) + 3(-3)$

$\Rightarrow -3 + 12 - 9$

$\Rightarrow 0$

$$\Delta' = 1 \begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} - 4 \begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} + 7 \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$$

$\Rightarrow 1(45-48) - 4(18-24) + 7(12-15)$

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$\Rightarrow 1(-3) - 4(-6) + 7(-3)$

$\Rightarrow -3 + 24 - 21$

$\Rightarrow 21 - 21 = 0$

$\therefore \Delta = \Delta'$ prooved

property-II

The value of determinat will not change, if any two rows or columns of a determinant are inter-changed, the sign of the determinant is Changed.

E.g.

$$\Delta = \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix} \text{ and } \Delta' = \begin{vmatrix} 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \end{vmatrix}$$

proof $\Delta = \Delta'$

Sol:

$$\Delta = 1\begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} - 4\begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} + 7\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$$

$\Rightarrow \Delta \Rightarrow 1(45-48) - 4(18-24) + 7(12-15)$

$\Rightarrow 1(-3) - 4(-6) + 7(-3)$

$\Rightarrow -3 + 24 - 21$

$\Rightarrow 0$

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$$\Delta' = 7\begin{vmatrix} 5 & 2 \\ 6 & 3 \end{vmatrix} - 4\begin{vmatrix} 8 & 2 \\ 9 & 3 \end{vmatrix} + 1\begin{vmatrix} 8 & 5 \\ 9 & 6 \end{vmatrix}$$

$\Rightarrow 7(15-12) - 4(24-18) + 1(48-45)$

$\Rightarrow 7(3) - 4(6) + 1(3)$

$\Rightarrow 21 - 24 + 3$

$\Rightarrow 21 - 21 = 0$

$\therefore \Delta = \Delta'$ proved

property - III

when ~~one or~~ two columns and ~~one or~~ two rows of a determinat are equal, the ~~the~~ value of that determinant will be zero.

e.g. $\Delta \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$

Sol

$$\Delta = 1\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix}$$

$\Rightarrow 1(3-4) - 2(1-1) + 1(4-3)$

$\Rightarrow 1(-1) - 2 \times 0 + 1 \times 1$

$\Rightarrow \cancel{1} - 0 + \cancel{1} = 0$ $\therefore \Delta = 0$ proved

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e.g 2

$$\Delta \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

prove

Sol 2

$$\Delta = 1 \begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix}$$

$\Rightarrow 1(15-12) - 2(12-6) + 3(8-5)$

$\Rightarrow 1 \times 3 - 2 \times 6 + 3 \times 3$

$\Rightarrow 3 - 12 + 9 = \cancel{9+9}$

$\Rightarrow 0$

$\therefore \Delta = 0$ proved

property IV

If each element of any row or column of a determinant is the sum of two quantities, then determinant can be expressed as the sum of two determinants.

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Eg

$$\begin{vmatrix} 16 & 15 & 14 \\ 12 & 10 & 9 \\ 8 & 6 & 4 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 10+6 & 15 & 14 \\ 10+2 & 10 & 9 \\ 4+4 & 6 & 4 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 10 & 15 & 14 \\ 10 & 10 & 9 \\ 4 & 6 & 4 \end{vmatrix} + \begin{vmatrix} 6 & 15 & 14 \\ 2 & 10 & 9 \\ 4 & 6 & 4 \end{vmatrix}$$

$$\Rightarrow 10\begin{vmatrix} 10 & 9 \\ 6 & 4 \end{vmatrix} - 15\begin{vmatrix} 10 & 9 \\ 4 & 4 \end{vmatrix} + 14\begin{vmatrix} 10 & 10 \\ 4 & 6 \end{vmatrix}$$

$\Rightarrow 10\cdot(40 - \cancel{54}) - 15(\cancel{400}\,40 - 36) + 14(60 - 40)$

$\Rightarrow 10\times 94 - 15\times 76 + 14$

$\Rightarrow 10\times(-14) - 15(-4) + 14(20)$

$\Rightarrow -140 + 60 + 280 \quad \Rightarrow 440 + 340 = 200$

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property - V

If a factor is common in any row or column then that common factor can be taken out as a product to the determinant.

e.g.

$$\begin{vmatrix} 2 & 15 & 8 \\ 4 & 12 & 3 \\ 8 & 16 & 4 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 15 & 8 \\ 2 & 12 & 3 \\ 4 & 16 & 4 \end{vmatrix}$$ (∵ taking 2 as a common from C_1)

$$= 2 \times 4 \begin{vmatrix} 1 & 15 & 8 \\ 2 & 12 & 3 \\ 1 & 4 & 1 \end{vmatrix}$$ (∵ taking 4 as common from R_3)

$$= 8 \times 1 \begin{vmatrix} 12 & 3 \\ 4 & 1 \end{vmatrix} - 15 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + 8 \begin{vmatrix} 2 & 12 \\ 1 & 4 \end{vmatrix}$$

$= 8(12-12) - 15(2-3) + 8(8-12)$

$= 8 \times 0 - 15 \times (-1) + 8 \times (-4)$

$= 0 + 15 - 32$

$= \cancel{20} - 17$

Q $\begin{vmatrix} 2 & 8 & 14 \\ 4 & 10 & 12 \\ 6 & 16 & 18 \end{vmatrix}$

Sol $\Rightarrow 2\begin{vmatrix} 10 & 12 \\ 16 & 18 \end{vmatrix} - 8\begin{vmatrix} 4 & 12 \\ 6 & 18 \end{vmatrix} + 14\begin{vmatrix} 4 & 10 \\ 6 & 16 \end{vmatrix}$

$\Rightarrow 2\{180-192\} - 8\{72-72\} + 14\{64-60\}$

$\Rightarrow 2 \times (-12) - 8 \times 0 + 14 \times 4$

$\Rightarrow -24 - 0 + 56$

$\Rightarrow -24 + 56$

$\Rightarrow 32.$

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*Q3

$$\begin{vmatrix} 14 & 16 & 18 \\ 8 & 10 & 12 \\ 2 & 4 & 6 \end{vmatrix}$$

Sol

$$+14\begin{vmatrix} 10 & 12 \\ 4 & 6 \end{vmatrix} - 16\begin{vmatrix} 8 & 12 \\ 2 & 6 \end{vmatrix} + 18\begin{vmatrix} 8 & 10 \\ 2 & 4 \end{vmatrix}$$

$\Rightarrow +14\{(10\times6) - (12\times4)\} - 16\{(8\times6) - (12\times2)\} + 18\{(8\times4) - (10\times2)\}$

$\Rightarrow +14(60 - \cancel{20}\ 48) - 16(48 - 24) + 18(32 - 20)$

$\Rightarrow +14 \times 12 - 16 \times 24 + 18 \times 12$

$\Rightarrow 168\ \cancel{504} - 384 + 216$

$\Rightarrow \cancel{720 - 384}\ 384$

$\Rightarrow \cancel{336}\ 384 - 384$

$\Rightarrow 0$

*Q4

$$\begin{vmatrix} 2 & 5 & 6 \\ 7 & 9 & 1 \\ 3 & 8 & 4 \end{vmatrix}$$

Sol

$$+2\begin{vmatrix} 9 & 1 \\ 8 & 4 \end{vmatrix} - 5\begin{vmatrix} 7 & 1 \\ 3 & 4 \end{vmatrix} + 6\begin{vmatrix} 7 & 9 \\ 3 & 8 \end{vmatrix}$$

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$\Rightarrow +2\{(9\times4)-(8\times1)\}-5\{(7\times4)-(3\times1)\}+6\{(7\times8)-(9\times3)\}$

$\Rightarrow +2(36-8)-5(28-3)+6(56-27)$
$\Rightarrow +2\times28-5\times25+6\times29$
$\Rightarrow 56-125+174$
$\Rightarrow 230-125$
$\Rightarrow 105$

*Q5 $\begin{vmatrix} 3 & 8 & 4 \\ 2 & 9 & 1 \\ 2 & 5 & 6 \end{vmatrix}$

~~$+3\{(9\times6)-5\times1$~~

Sol $+3\begin{vmatrix} 9 & 1 \\ 5 & 6 \end{vmatrix} - 8\begin{vmatrix} 7 & 1 \\ 2 & 6 \end{vmatrix} + 4\begin{vmatrix} 7 & 9 \\ 2 & 5 \end{vmatrix}$

$\Rightarrow +3\{(9\times6)-(5\times1)\}-8\{(7\times6)-(2\times1)\}+4\{(7\times5)-(9\times2)\}$

$\Rightarrow +3(54-5)-8(42-2)+4(35-18)$
$\Rightarrow +3\times49-8\times40+4\times17$
$\Rightarrow 147-320+68$
$\Rightarrow 215-320$
$\Rightarrow -105$